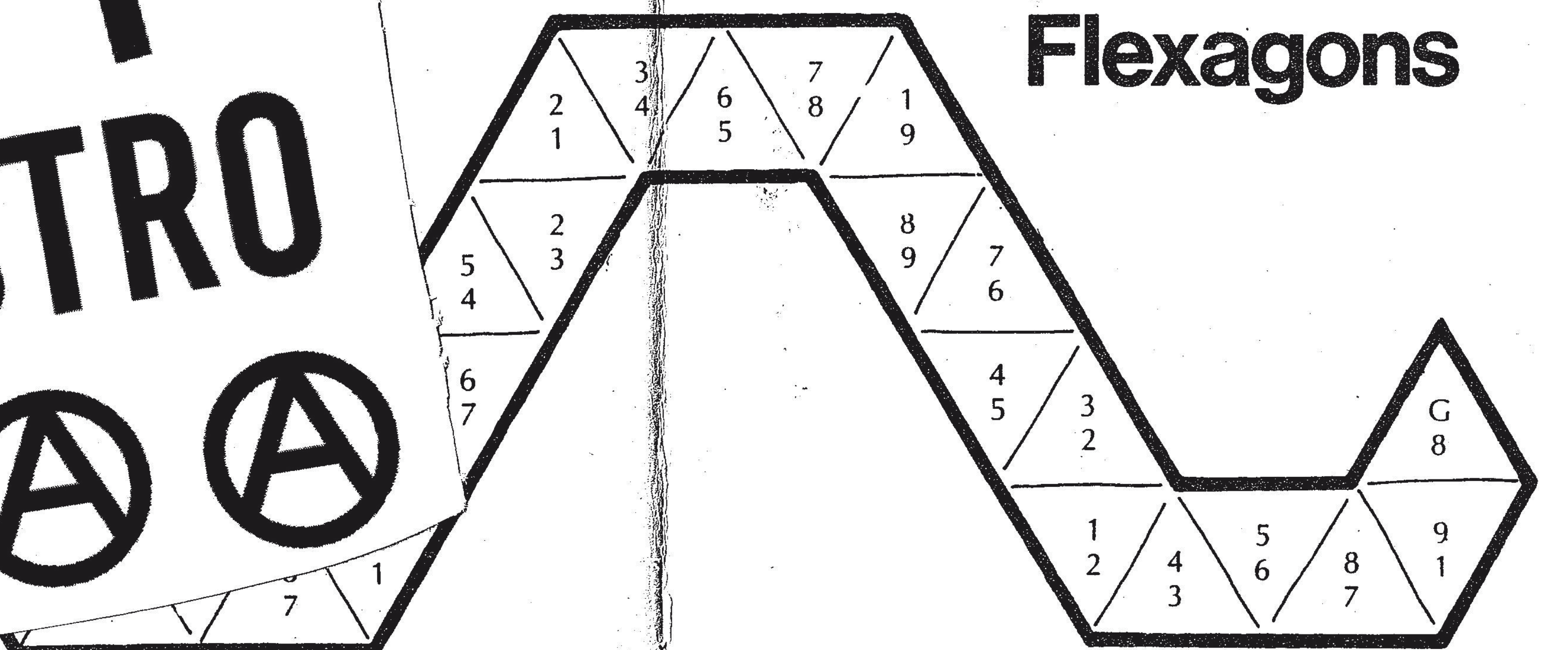
BRITISH ORIGAMI SOCIETY

Flexagons



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Paul Jackson

FTP DISTRO

Flexagons

Paul Jackson

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Flexagons

Paul Jackson

British Origami Society

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47	ABC of Origami	Eric Kenneway
48	Making Faces	David Petty

BIBLIOGRAPHY

Very few books have been written about flexagons, and most magazine articles say the same things about hexaflexagons. However, here is a list of some of the more interesting publications.

Bolt, R. 'Magic With Origami', B.O.S. Booklet No. 9
Contains a hexaflexagon, the fascinating 'Inside-out Cube' and Robert Neale's 'Sheep and Goats'.

Gardner, M. Mathematical Puzzles and Diversions', and 'More Mathematical Puzzles and Diversions', Pelican.
The former contains an interesting chapter on hexaflexagons, the latter on tetraflexagons.

Jones, M. 'Mysterious Flexagons', Crown Publishers Inc. New York.
Contains hexa and tetraflexagons.

Kenneway, E. 'Action Origami', Dryad Press.
Contains Iris Walker's flexagon.

Madachy, S. 'Mathematics on Vacation', Nelson.
Contains the hexaflexagon formula.

Oakley, C. O. and Wisner, R. J. 'Flexagons. 'American Mathematical Monthly', Vol. XVI, March 1957, pages 143–154.
This is a mathematical analysis of hexaflexagon structure.

The following are on loan from the B.O.S. Library

Sheep and Goats–MO95
Strip Flexagon–J424
Iris Walker's Ring Flexagon–MO32
Cross Flexagon (first published in Gardner's 'Scientific American' column)–J536
Flexicube–M240

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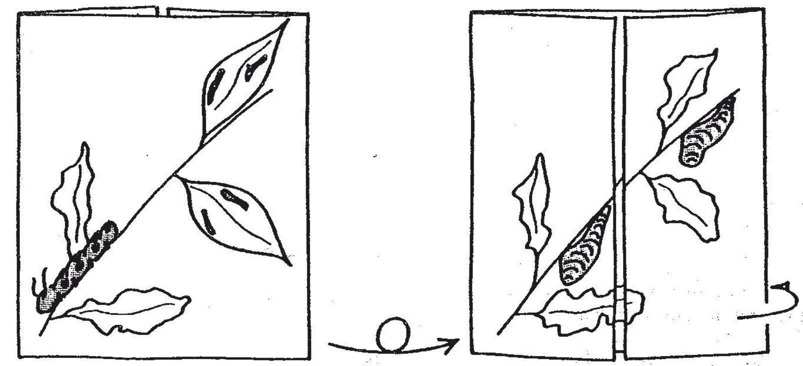
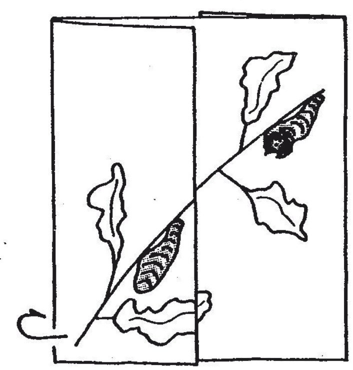


The caterpillars grow bigger, eating up the leaves . . .

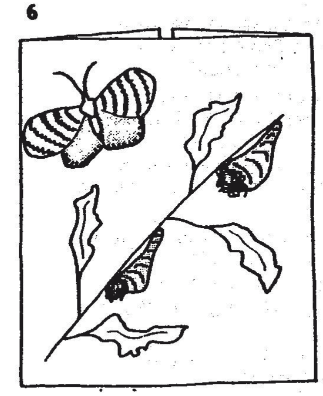
. . . and then turn into pupae

5



After a few weeks, the pupae open . . .

6



. . . and a butterfly emerges

Finally, flexagons can be used whenever an idea requires a sequence of images or words. One idea, the story, has been dealt with above– others include: greetings cards, gift tags, jokes, sequential puzzles and calendars.

Have fun!

2 COLOURS, PATTERNS AND PICTURES

Throughout the booklet I have given a number to each face revealed in a flexagon. This was done for simplicity and continuity. Many flexagons however, are at their most entertaining when faces are coloured or patterned instead of numbered. They look more attractive, and often help reveal intriguing oddities within the flexing movements–hexaflexagons being a good example of this.

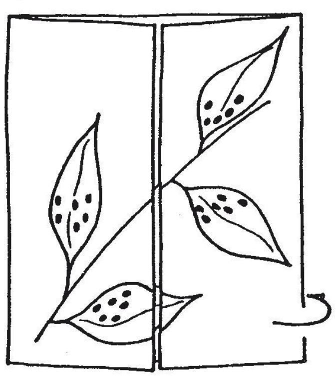
When demonstrating a flexagon with many faces, an audience isn't always convinced that the face just revealed is a different colour or pattern to one shown earlier in the sequence. An obvious way out is to number each face, but a lot depends on your audience. Children for example, prefer to look at bright colours rather than a lot boring numbers.

Another idea is to put a picture onto each face, either as a drawing or a photograph. This way, a flexagon can be given a theme.

Instead of a theme, a story could be constructed around the pictures. Ideally, a flexagon should be found to fit the story, rather than the other way around. Of all the ways to mark a face, this is probably the most open ended and least explored.

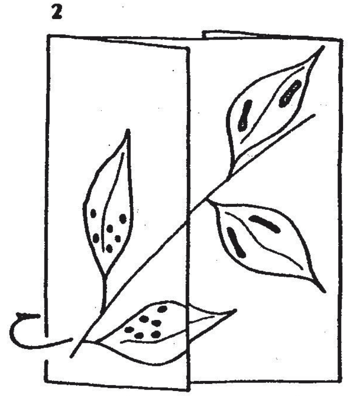
Philip Noble has produced a simple story flexagon, adapted from an idea by Sieshiro Yuasa, which relates the life cycle of a butterfly.

1



Small black eggs on the underside of leaves . . .

2



. . .hatch into caterpillars

INTRODUCTION

Flexagons are two or three dimensional configurations of paper which, when flexed, bring internal faces to the surface, hiding those previously in view.

They were first discovered in 1939 by an English Postgraduate Mathematics student at Princetown University called Arthur Stone. He had to trim his American notepaper to make it fit his smaller English binder, and whilst playing with the strips of paper that he had cut off, he accidentally folded the first flexagon.

Early research by Stone and his colleagues tended to concentrate on the mathematical structure of hexaflexagons. More recently, puzzlers and paper-folders have invented many others primarily for entertainment, not analysis.

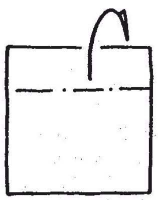
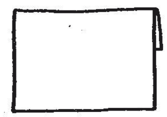
All flexagons contain fascinating oddities within their structure or flexing movements. Lack of space has prevented me from detailing all of the known ones. Therefore, after you have made a flexagon, play around with it, folding it this way and that to see what happens. Such experimentation could lead you to invent your own flexagons–and don't be surprised if you do, because the subject is little understood and there are large areas still to be explored (for example, flexagons which change their shape when flexed instead of revealing new faces).

Finally, can I apologise for the long, almost unintelligible names that some flexagons have been given. Such names are mathemetical descriptions, and have unfortunately become standard.

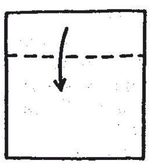
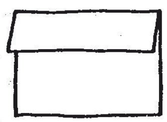
Symbols

For the benefit of those non paper folders who may read this booklet, here is an explanation of the standard origami symbols used within.

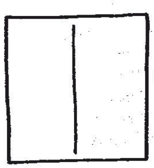
 means make a mountain fold

thus  means fold behind 

 means make a valley fold

thus  means fold in front 

 turn over

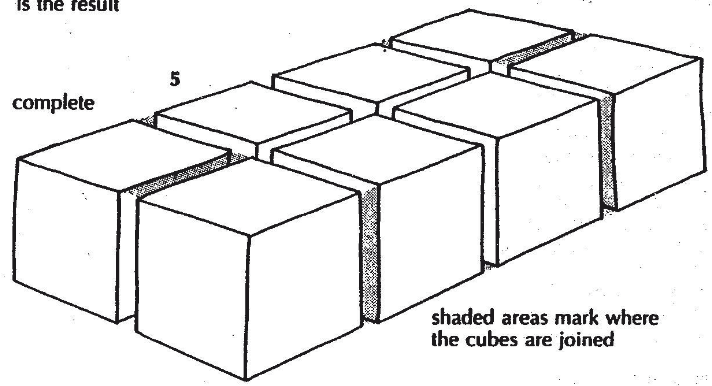


a continuous thin line shows an existing crease

Begin with a 130 × 1 strip of paper and make a cube with 16 squares to one side of the centre of the strip. At the other side of the centre, make another cube which is the mirror image of the first. Continue away from the centre on either side, making pairs of mirror image cubes. Feed the loose tabs at the ends of the strip into adjacent cubes to lock the flexagon. Here is the result

5

complete



shaded areas mark where the cubes are joined

A simpler alternative is to make 8, 17 × 1 cubes and feed the loose tab formed by the additional 17th square into an adjacent cube to lock them together. Eight cubes made from wood, metal etc. can also be used if taped together.

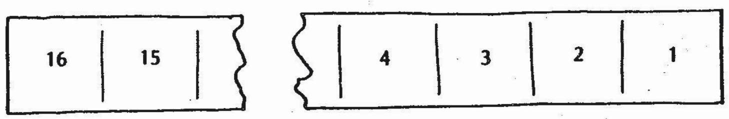
Flexing

The sequence is obvious so I won't draw it up.

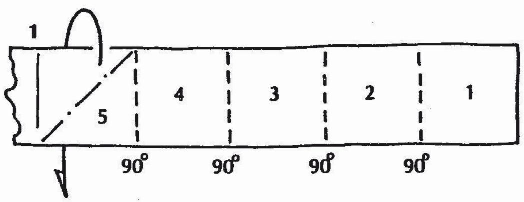
OTHER IDEAS

1 FLEXICUBE–Philip Noble

Begin with a 16 × 1 rectangle, numbered and creased as follows:



1



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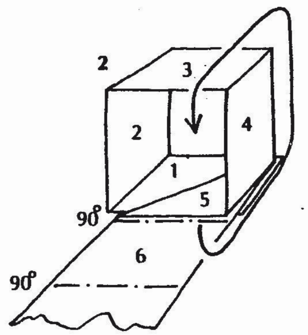
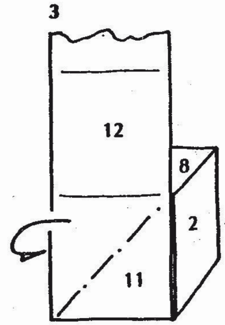


wrap 6 to 11 around the cube

3



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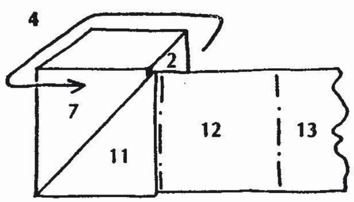


12 is fed under 2
13 goes over 9
14 is fed under 4
15 goes over 7/11
16 is fed under 2
This locks the cube

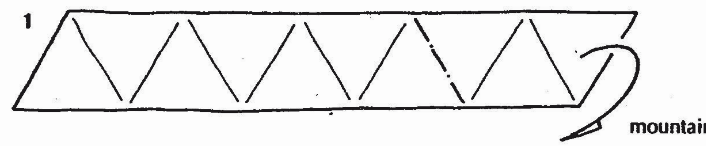
HEXAFLEXAGONS

1 TRI-HEXAFLEXAGON

This is the first flexagon discovered by Stone, and is probably the simplest that can be made.
Begin with a strip of ten equilateral triangles made from strong paper.

Construction

1



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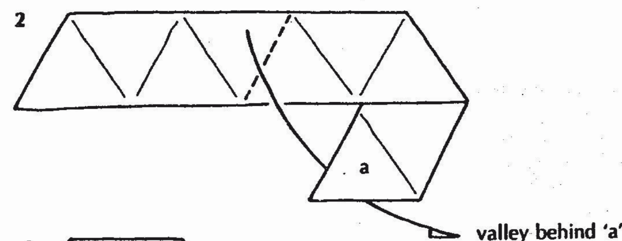
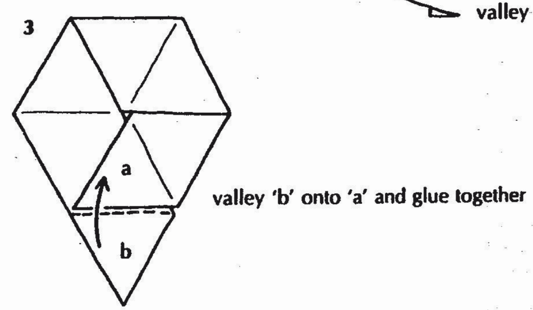
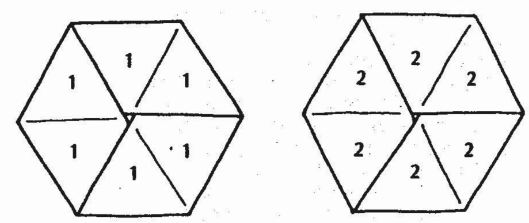


3

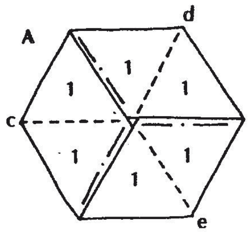


number as shown



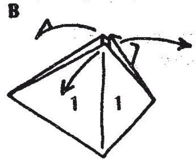
Flexing

A



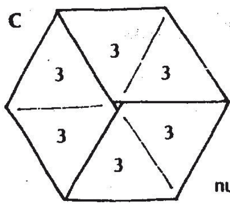
bring points 'c,d,e,' together

B



open out at the top

C



number the new face

Face 2 has disappeared and 3 has taken its place. A further flex will make 1 disappear and bring 2 and 3 together.

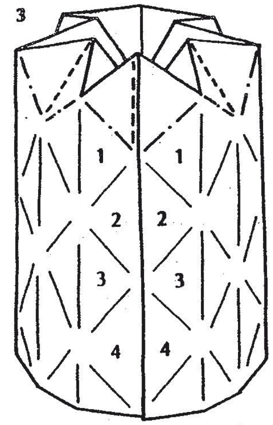
2 THE COVER FLEXAGON

The illustration on the cover shows an unfolded hexaflexagon with nine faces.

Construct the strip from strong paper, letting the sides of each triangle be about 4cms. long. The top number in each triangle is to be written on the front of that triangle, the bottom number on the reverse. This principle will apply throughout the booklet.

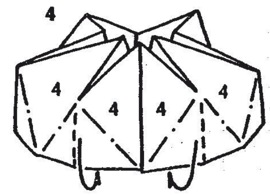
The strip is then folded up as described on page 14. The two 'G's are glued together to lock the resulting hexagon, much as in diagram 3 on page 7. Similarly, the flexing movement is the same as described above. When flexed, faces 1 to 9 will reveal themselves in sequence.

3



Continue to push the diamonds down through the centre, until diagram 4 is reached

4

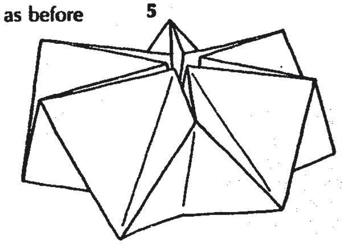


push in the remaining triangles

complete

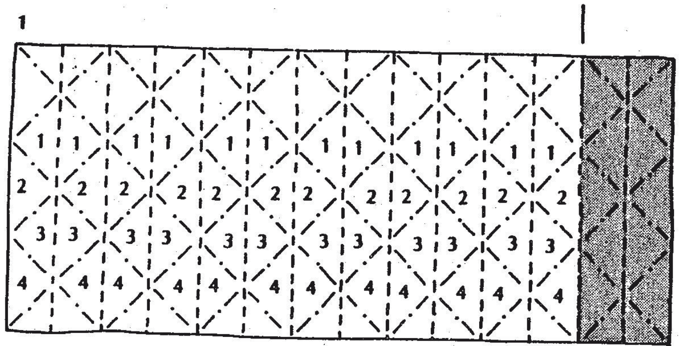
flex as before

5



Pre-crease and number a 7 × 3 ·ectangle as follows:

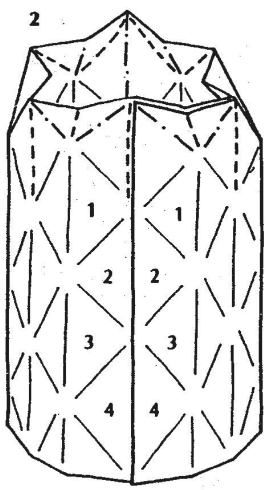
1



for overlapping

Fold the above into a cylinder with the numbers on the outside. Let the left-hand edge overlap the shaded portion on the right

2



Push in the six triangles at the top, then push in the six diamond shapes which lie between them

Hexaflexagons can be made to flex to any number. There are published examples of hexa-hexaflexagons (6 faces) and a few others, but little that is comprehensive. Perhaps the most effective way of listing such flexagons is not to draw out a lot of unfolded strips, such as the one on the cover of this booklet, but to give a formula so that those who are interested can work them out for themselves. This formula was invented by Stone and has been adapted and simplified by myself for this booklet. Although long, it is not complicated or difficult. Please follow the instructions with special care.

Part A

Draw a polygon which has the same number of sides as you want your flexagon to have faces.

Number each corner, travelling in a clock or anti-clockwise direction starting anywhere.

Join the corners up in such a way that no two lines cross. There are many variations for each polygon.

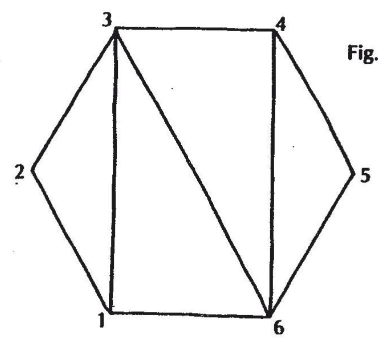
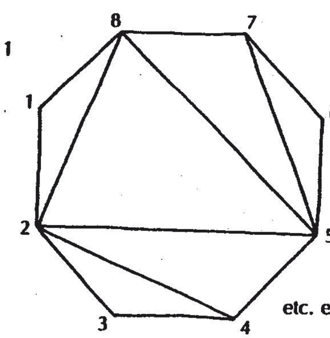


Fig. 1

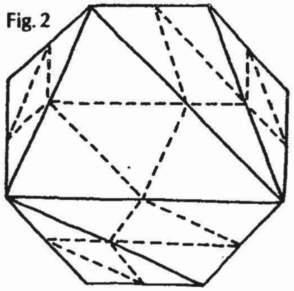


etc. etc.

Note that some corners will be left unconnected

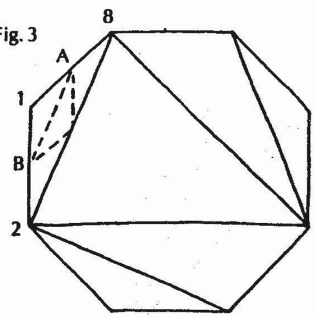
Divide each side of the polygon in half, and divide each internal line in half. Join up these points as in Fig. 2, creating a series of dashed triangles.

Fig. 2



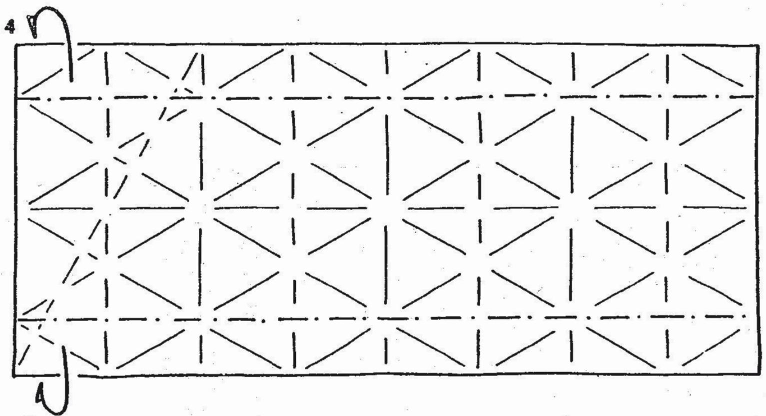
Choose a Fig. 1 triangle which has two of its sides as edges of the polygon (e.g. triangle 1,2,3 in the second Fig. 1 diagram). On this triangle, write 'A' and 'B' where the corners of the dashed triangle touch the polygon. Write them travelling in a **anti-clockwise** direction (Fig. 3).

Fig. 3



Follow the dashed path from 'A' to 'B' and then to the inside corner of the dashed triangle. Everytime the dashed path meets three others, go straight through, ignoring paths to the left and right. Eventually, the path will emerge on another side of the polygon. Mark this 'C'. Follow the dashed path in a similar way until all the sides have been marked in alphabetical order and the path has arrived back at 'A'. See Fig. 4.

4



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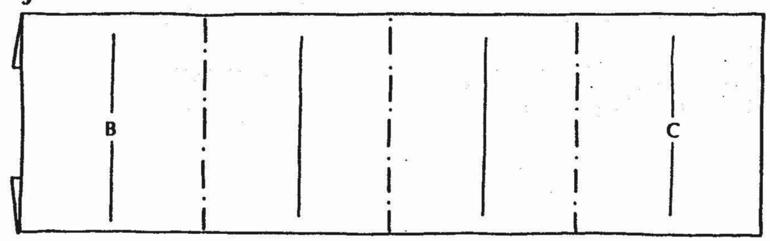

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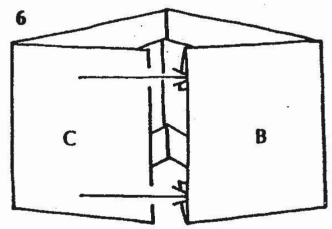
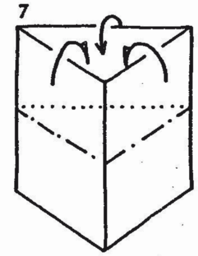


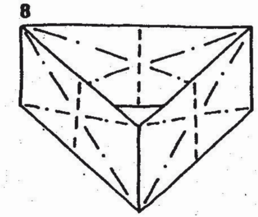
tuck 'C' into 'B'

7



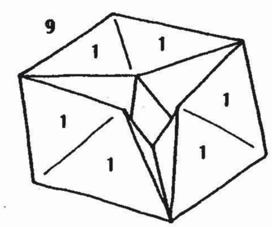
mountain inside
all the way
round (not easy!)

8



push in the three sides,
top and bottom

9

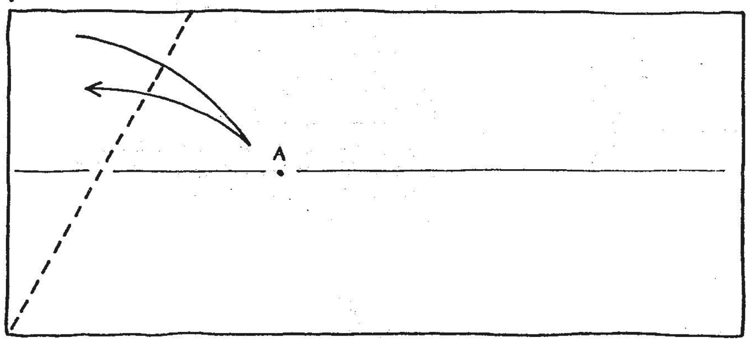


number and flex as before

making this flexagon from a soft paper helps it to flex smoothly. A hard paper can make the movement stiff

Begin with a dollar bill, or a 16 × 7 rectangle.

1



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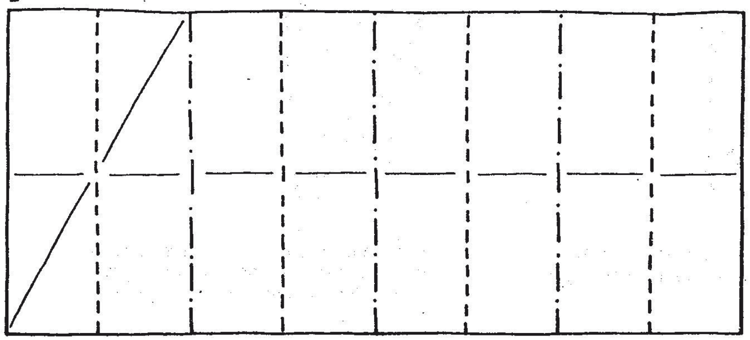

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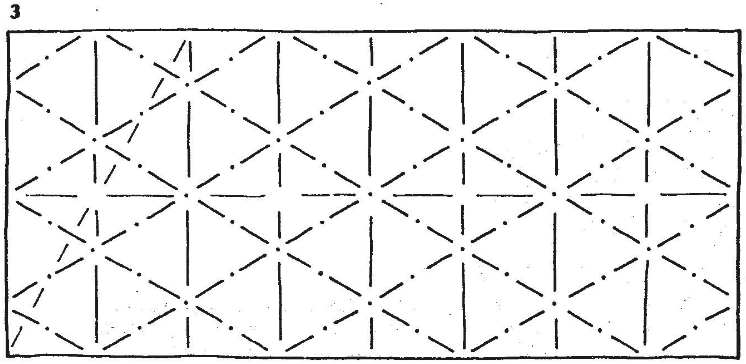
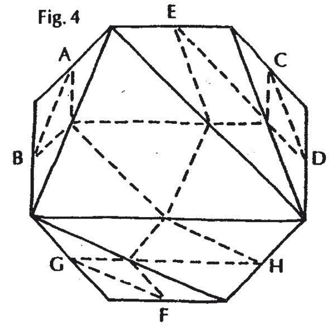
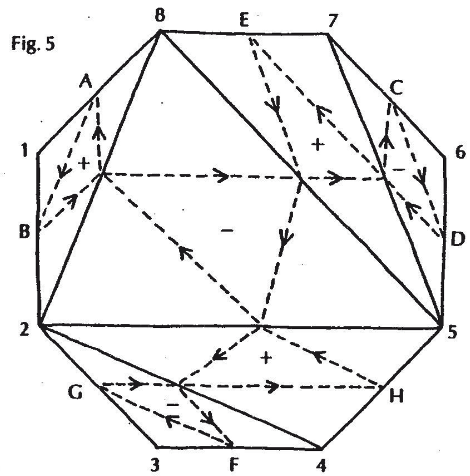


Fig. 4



Each dashed triangle is traversed in either a clockwise or an anti-clockwise direction whilst producing Fig. 4. If that direction is anti-clockwise write a '+' inside that triangle. If the direction is clockwise, write a '−'. See Fig. 5.

Fig. 5



Part B

Write out each letter used in the polygon
A B C D E F G H

Beneath A,C,E . . . (alternate letters) write the number which is on the positive (anti-clockwise) side of that letter on the perimeter of the polygon. For B,D,F . . . write the number on the negative (clockwise) side

A B C D E F G H
1 1 7 5 8 3 3 4

Beneath this write a third line which is the opposite of line two. Therefore, write down the numbers on the negative side of A,C,E . . . and those on the positive side of B,D,F . . .

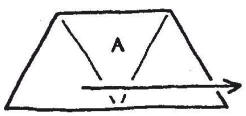
A B C D E F G H
1 1 7 5 8 3 3 4
8 2 6 6 7 4 2 5

Beneath this write the '+' or '−' symbol of the dashed triangle of which the appropriate letter forms a corner

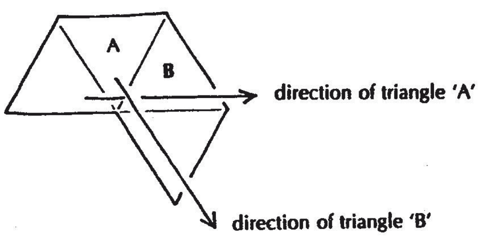
A B C D E F G H
1 1 7 5 8 3 3 4
8 2 6 6 7 4 2 5
+ + − − + − − +   Fig. 6

Part C

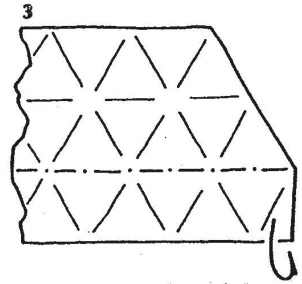
Three equilateral triangles in a line give the centre one 'A' a direction



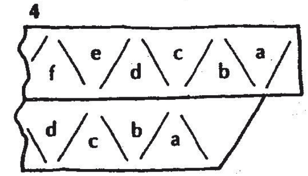
Three equilateral triangles in a line ('A','B' and blank) give 'B' a different direction to that of 'A'



3

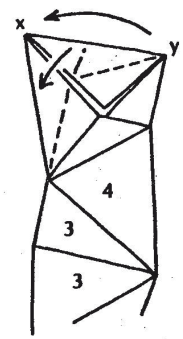


4



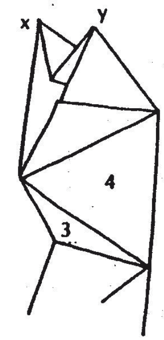
Fold 'a' onto 'a', 'b' onto 'b' etc. down the strip. This makes a series of connected tetrahedrons

5



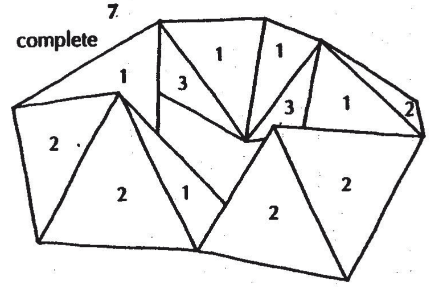
Flatten the unnumbered tetrahedron, which was the last to be formed. Let 'x' touch 'y'

6



Insert into the slit between faces 2 and 4 at the other end of the strip. This locks the tetrahedron into a ring

7



Flexing

Roll the faces either towards or away from the centre, so that as one number disappears, another comes into view.

RING FLEXAGONS

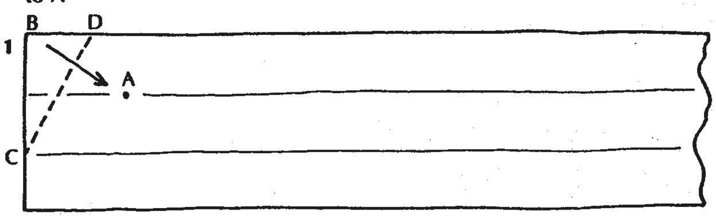
These flexagons have probably been around for centuries, since tetrahedrons can easily be made from materials such as wood or metal, and the obvious thing to do with three or four of them is to link them into a ring. Only recently have paper-folders found a quick and pretty way of doing the same.

Iris Walker's flexagon is the only non-tetrahedron ring that I know of. No doubt there are others waiting to be discovered.

1 ROBERT NEALE'S

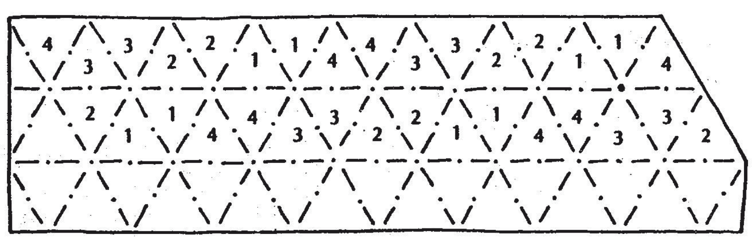
Crease a rectangle which is about 4 × 1 as shown. Angle B,C,D is 60° when folded to A

1

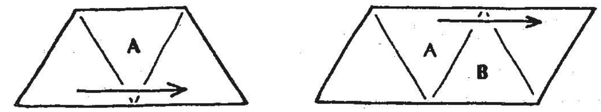


Using this crease as a guide, fold a series of equilateral triangles along the strip. Trim the strip at the correct point shown below, and number

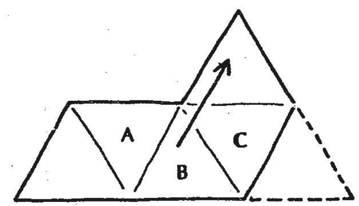
2



In Fig. 6, the bottom row consists '+' and '−' signs. 'A' and 'B' have '+' signs. This means that these two triangles have the **same** direction

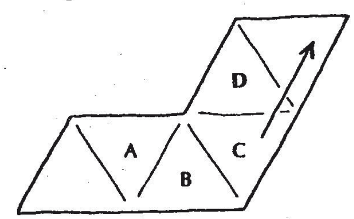


'C' is a '−' and so **changes** the direction

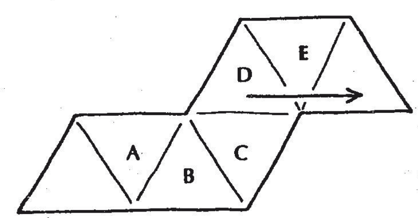


position of blank triangle if 'C' had the same direction as 'A' and 'B'

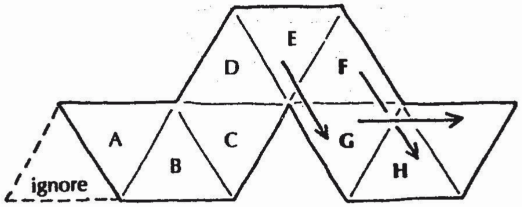
'D' is also a '−' and maintains that change



'E' is a '+' sign and so changes the direction



'F' and 'G' are '–' signs and so change and maintain that change in direction. 'H' is a '+' and changes the direction



Repeat this pattern three times and put a fourth 'A' at the end. 'H' to 'A' is not a change in direction ('+' to '+').

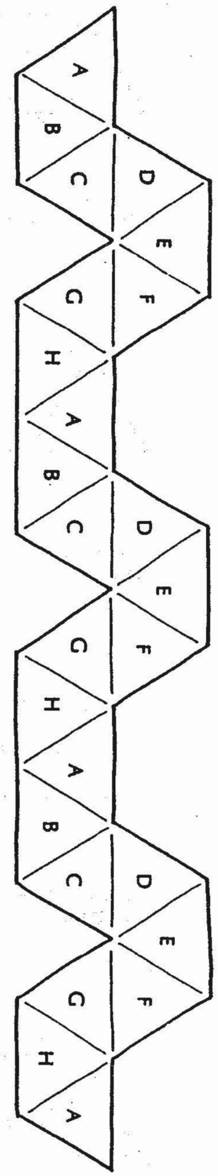
Part D

If the original polygon (Fig. 1) has an even number of sides, number the triangles as follows:
Write the second row of Fig. 6 in order, on one side of Fig. 7 three times. The third row numbers go on the reverse side. The two numbers for triangle 'A' are on the same triangle in Fig. 7, the same for 'B', 'C' etc.

If the original polygon has an odd number of sides, number them as follows: On one side of the strip, write the second row of numbers, followed by the third row of numbers, followed by the second row of numbers, followed by the third row 'A'. On the other side, write the third row, second row, third row and second row 'A' in order, starting at the same triangle as the first side was begun at. Fig. 7 can now be numbered, numbers below go on the reverse side.

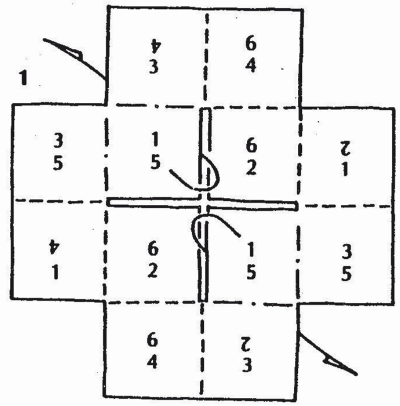
Cut out this shape from strong paper, making the side of each triangle about 4 cms. long.

To fold up, simply fold adjacent triangles with like numbers against each other, hiding the like numbers from view. This does not apply to the triangles at the ends of the strip. Continue this process until the final hexagon is obtained. Glue the ends together and flex as described on page 8.

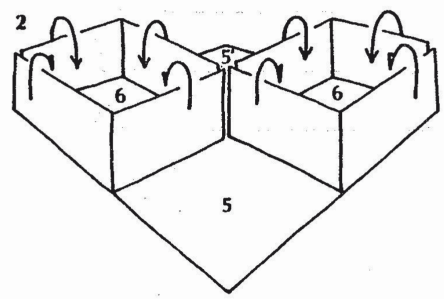


6 CROSS FLEXAGON–Robert Neale

Construction



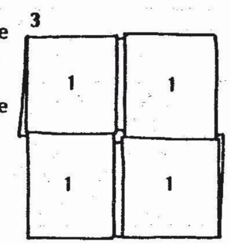
note the cut
make 3D, letting 'a's flip to the centre



Simultaneously push down on the inside corners of the two 5 squares. Continue to push, and the boxes flatten themselves.

complete

2's on the reverse side

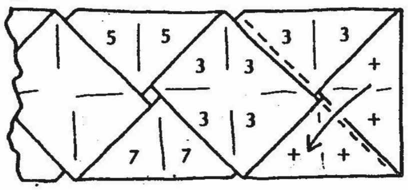


Flexing

Flex as for the Tri-tetraflexagon to find faces 1 to 4. Faces 5 to 6 have to be found by a different flexing movement which is difficult to discover. Dare you to find it!

Reverse the squashing sequence in diagrams A to G to return to diagram A. Notice that the 8 crosses that were together in diagram H have split into two groups of four at opposite ends of the strip!!(??)

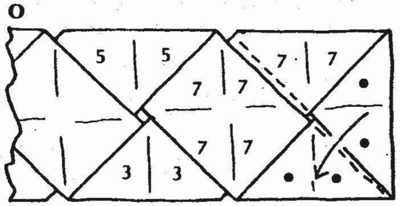
N



valley the crosses onto each other

Repeat diagrams A–G to return to diagram M. Reverse the flexing sequence H–M to return to diagram A. Notice that the eight circles that were together in diagram M have split into two groups of four at opposite ends of the strip.

O



valley the circles onto each other to repeat the cycle A to O

'Strip Flexagons' can be made which have many more faces than the one illustrated above. The only rule is that the plaited strip–diagram A–should be one unit wide by an odd number long (diagram A is 1 × 5).

There are some unmarked squares on the strip which I have been unable to flex into view. Is there a way?

Diagram 6 has an identical plaited pattern to diagram E in the second flexing method for the 'Double Square' flexagon. Both patterns are derived from the 'Sheep and Goats' flexagon/puzzle by Robert Neale (see Bibliography).

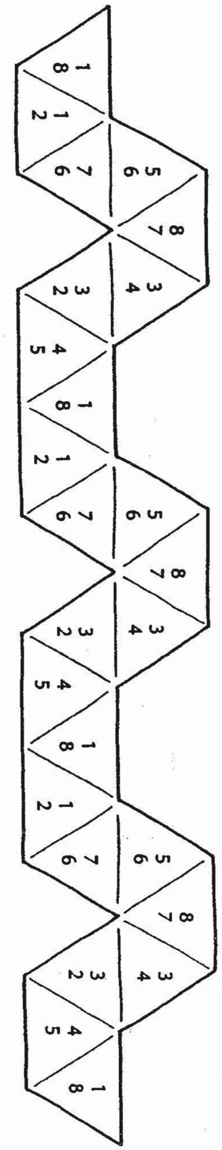


Fig. 7

note:
to make a hexaflexagon that flexes 1 . . 2 . . 3 . . 4 etc. in sequence, draw a polygon and radiate all internal lines from one corner to obtain Fig. 1.

TETRAFLEXAGONS

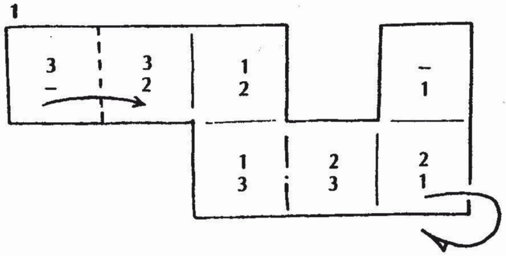
Tetraflexagons are similar to hexaflexa-gons. The major difference is that they are based upon 45° and 90° angles, instead of 60° and 120° angles, which alters thei· flexing geometry.
Here is the simplest.

1 TRI-TETRAFLEXAGON

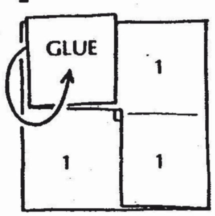
Construction

Cut out and number a piece of stiff paper as shown, the side of each square being about 4cms. long. The lower numbers in a square go on the reverse side of that square. Two squares have no numbers on one side.

1



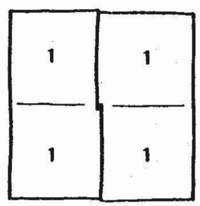
2



bring the rear square to the top and glue

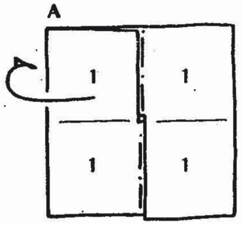
3

complete

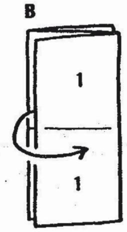


Flexing

A

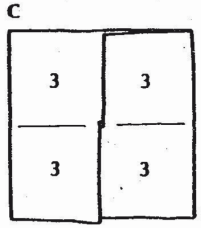


B

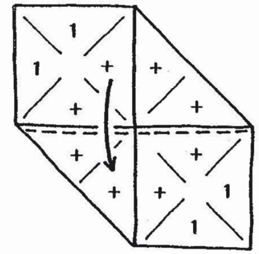


open out

C

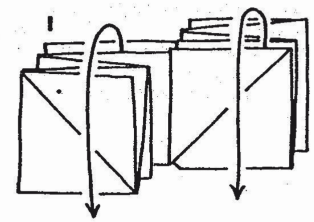


H

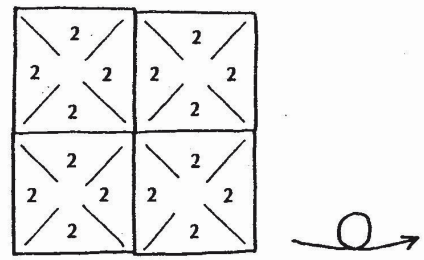


mark the flat side with '1's and crosses as shown, then valley

I

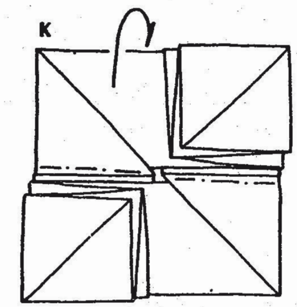


J



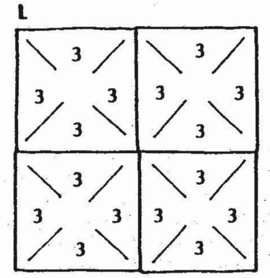
number the newly exposed face

K



mountain in half and open out again as in I.

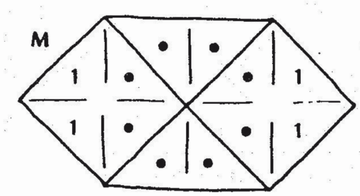
L



number the newly exposed face

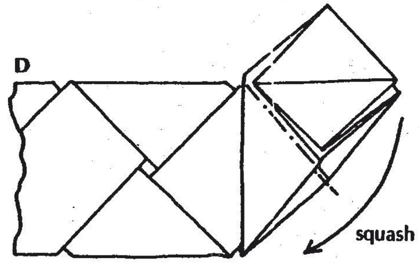
Continue to repeat the flexing movements in diagrams I–L, exposing faces 4–8. Number them. A further book fold will bring about a configuration identical to that in diagrams G and H. Turn over

M

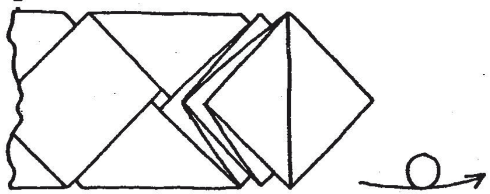


mark the vacant triangles with circles

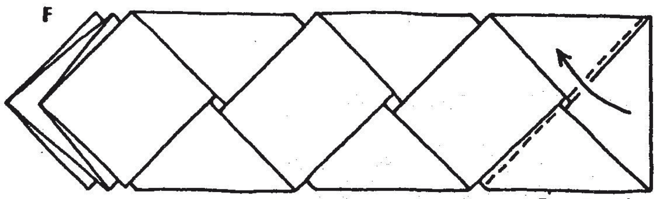
D



E

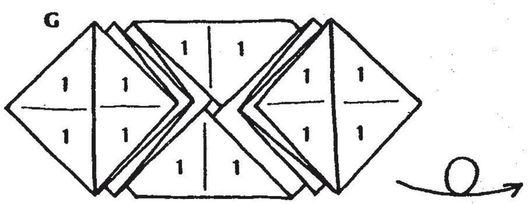


F



repeat B to E with this half of the strip

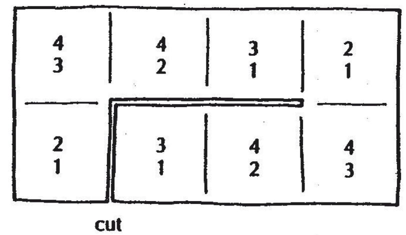
G



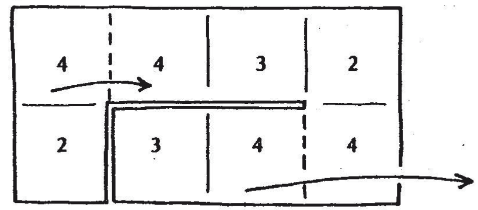
mark each small triangle with a '1'

2 TETRA-TETRAFLEXAGON

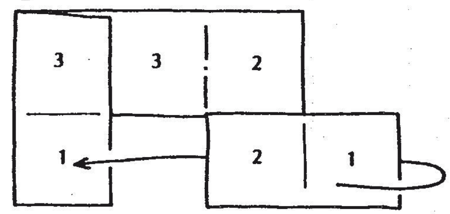
Construction



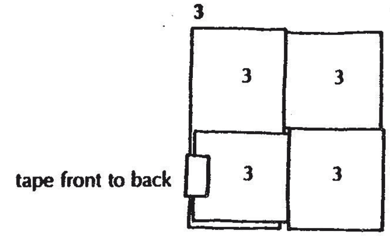
1



2



3

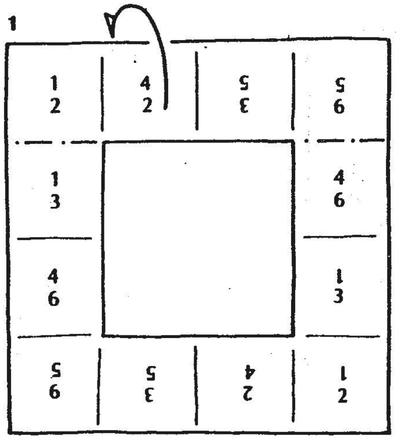


Flexing

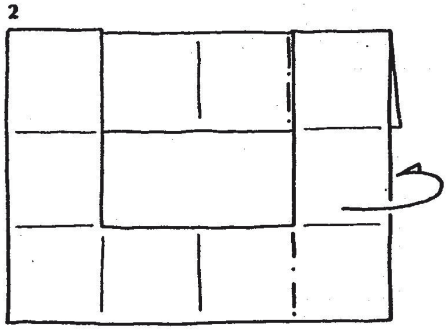
as for the tri-tetraflexagon

3 HEXA-TETRAFLEXAGON

1



2



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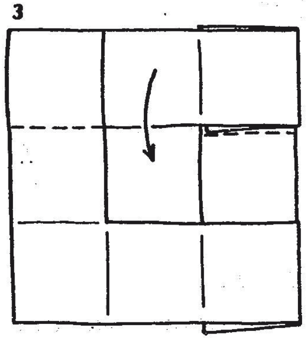

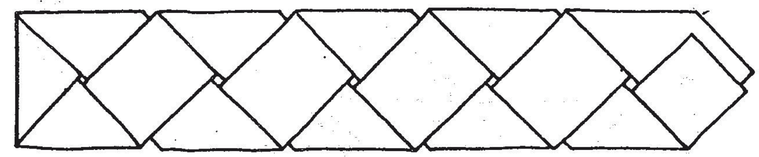

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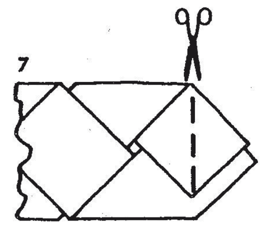
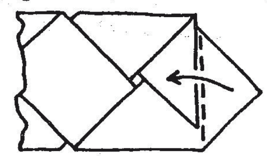
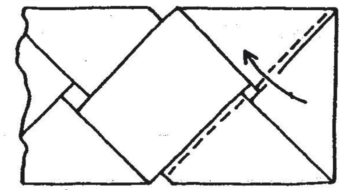


8

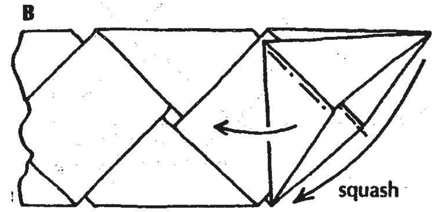


Flexing

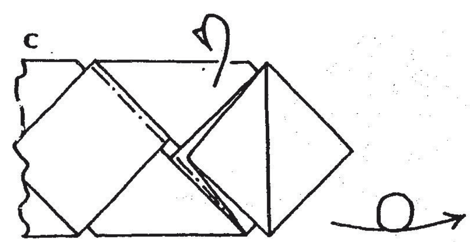
A



B

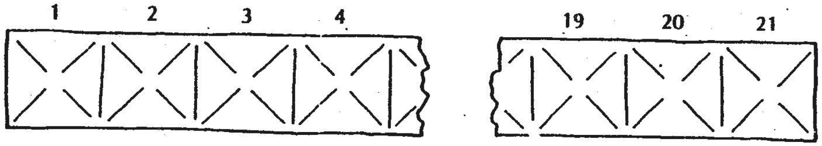


C



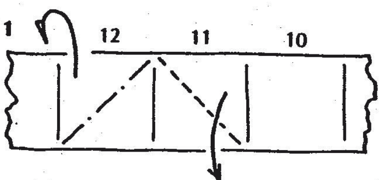
5 STRIP FLEXAGON—**Paul Jackson**

Begin with a strip of strong, thin paper 1" × 21" Crease as follows



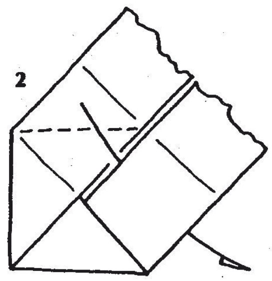
Construction

1



note the square numbers

2



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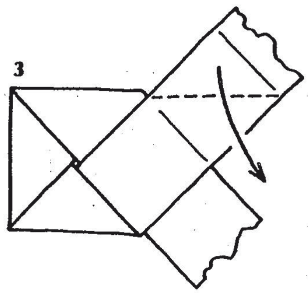
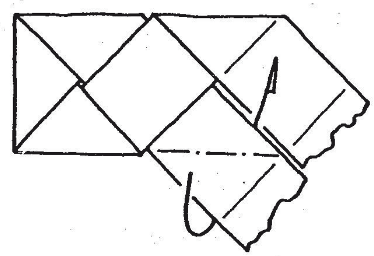
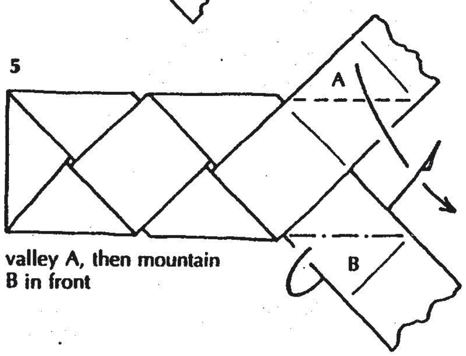


4



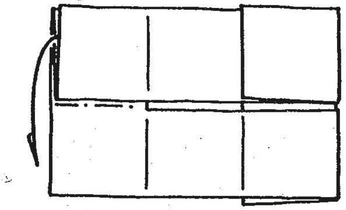
5



valley A, then mountain B in front

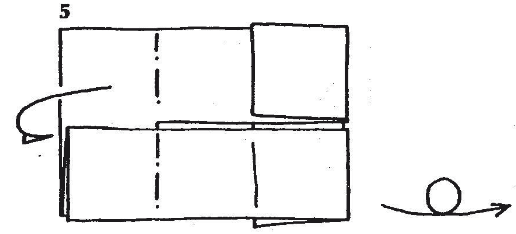
continue this sequence down the rest of the strip

4

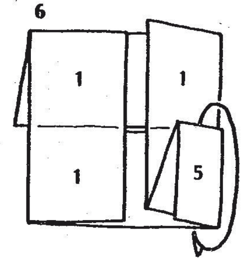


swivel rear flap downwards

5



6

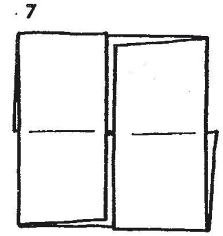


fold squares 5,6,6 and 4 to the rear, exposing the fourth square. This is difficult and involves some unfolding

complete

flex as before

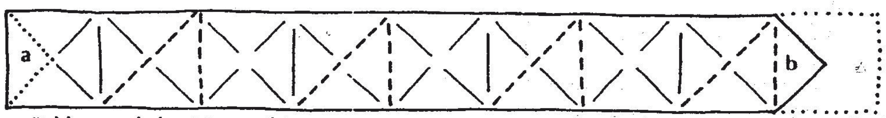
7



Flexagons similar to the one above can be made with many more faces. This suggests the existence of a formula for constructing them to a given number, much as there is a formula for constructing hexaflexagons. Can anybody find one?

4 DOUBLE SQUARE–**Paul Jackson**

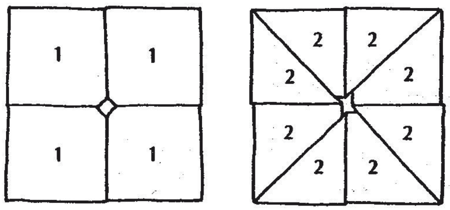
Begin with a 9 × 1 rectangle of strong, thin paper



Fold up and glue 'a' onto 'b'

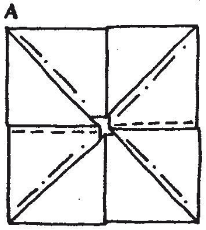
cut off

number as follows



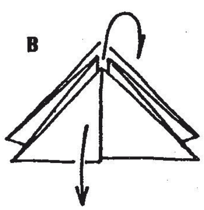
Flexing–method 1.

A



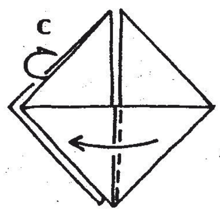
make a Waterbomb Base

B



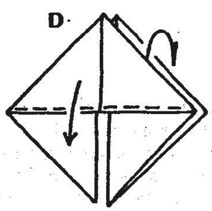
pull down front and back

C



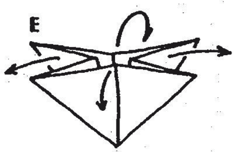
swivel front and back

D



pull down

E

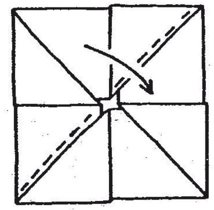


open out

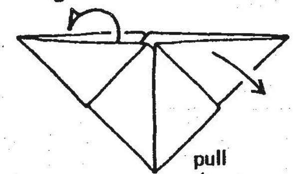
number the new faces

method 2

A

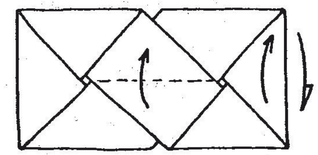


B



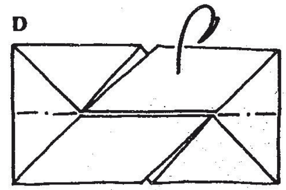
pull down

C

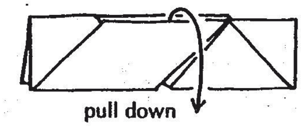


rotate

D

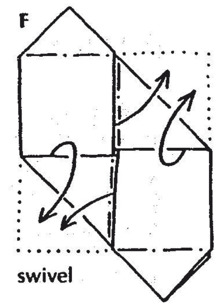


E



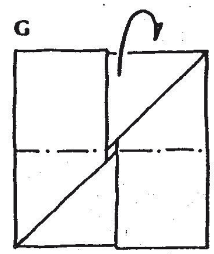
pull down

F

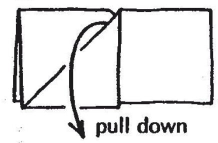


swivel

G

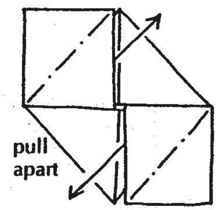


H



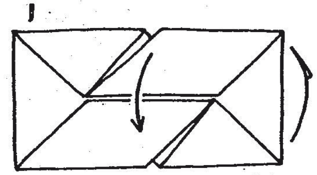
pull down

I



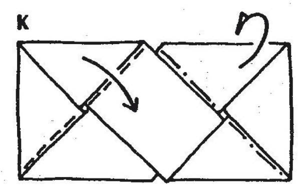
pull apart

J

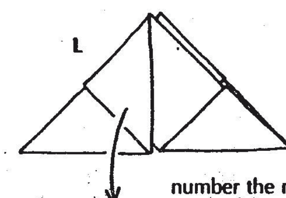


rotate

K



L



number the newly exposed faces

To return to faces 1 and 2, repeat 1st to 12th stages of flexing **in that order**